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18CS36

## Third Semester B.E. Degree Examination, Feb./Mar. 2022 Discrete Mathematical Structures

Time: 3 hrs.

Max. Marks: 100

*Note: Answer any FIVE full questions, choosing ONE full question from each module.*

### Module-1

- 1 a. Prove that for any propositions  $p, q, r$  the compound proposition  $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$  is a Tautology. (08 Marks)
- b. Prove the logical equivalence without using truth table:  
 $p \rightarrow (q \rightarrow r) \Leftrightarrow (p \wedge q) \rightarrow r$  (05 Marks)
- c. Find whether the following argument is valid. No engineering student of first or second semester studies logic.  
Anil is an Engineering student who studies logic  
 $\therefore$  Anil is not in second semester (07 Marks)

**OR**

- 2 a. Give a direct proof and an indirect proof for the given statement. "If 'n' is an odd integer, then  $n + 9$  is an even integer". (06 Marks)
- b. Prove the given logical equivalence problem using laws of logic.  
 $(p \rightarrow q) \wedge [\neg q \wedge (r \vee \neg q)] \Leftrightarrow \neg (q \vee p)$ . (07 Marks)
- c. Verify the given argument is valid or not?  
 $p \rightarrow (q \rightarrow r)$   
 $p \vee \neg s$   
q  
 $\therefore s \rightarrow r$  (07 Marks)

### Module-2

- 3 a. Prove that for each  $n \in \mathbb{Z}^+$   
 $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{1}{6} n(n+1)(2n+1)$  (07 Marks)
- b. Find the number of permutation of the letter of the word "MASSASAUGA". In how many of there all four 'A's are together? How many of them begin with 'S'? (06 Marks)
- c. Find how many distinct four digit integers one can make from the digit 1, 3, 3, 7, 7, 8. (07 Marks)

**OR**

- 4 a. Determine the co-efficient of  $xyz^2$  in the expansion of  $(2x - y - z)^4$ . (06 Marks)
- b. In how many ways can 10 identical pencils be distributed among 5 children in following cases:  
i) There are no restrictions.  
ii) Each child gets atleast one pencil.  
iii) The youngest child gets at least two pencils. (07 Marks)
- c. Find the number of arrangements of all the letters in "TALLAHASSEE"? How many of these arrangement have no adjacent 'A's'? (07 Marks)

**Module-3**

- 5 a. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be defined by
- $$f(x) = \begin{cases} 3x - 5 & \text{for } x > 0 \\ -3x + 1 & \text{for } x \leq 0 \end{cases}$$
- find  $f^{-1}(0), f^{-1}(1), f^{-1}(3), f^{-1}(-3), f^{-1}(-6), f^{-1}([-5, 5])$ . (07 Marks)
- b. On the set  $\mathbb{Z}^{-1}$  a relation 'R' is defined by  $aRb$  if and only if "a divides b (exactly)" verify that 'R' is equivalence relation. (06 Marks)
- c. Draw the Hasse diagram representing the positive divisor of 36. (07 Marks)

**OR**

- 6 a. Let  $A = \{1, 2, 3, 4, 5\}$  define relation 'R' on  $A \times A$  by  $(X_1, Y_1) R (X_2, Y_2)$  if and only if  $X_1 + Y_1 = X_2 + Y_2$ .
- i) Verify 'R' is a equivalence relation on  $A \times A$
- ii) Determine the partition of  $A \times A$  induced by R. (07 Marks)
- b. Let  $A = \{1, 2, 3, 4, 6\}$  and 'R' be a relation on 'A' defined by  $aRb$  if and only if "a is multiple of b" represent the relation 'R' as a matrix, draw its diagraph and relation R. (06 Marks)
- c. Let f, g, h be a function from  $\mathbb{R}$  to  $\mathbb{R}$  defined by  $f(x) = x + 2, g(x) = x - 2, h(x) = 3x$  for  $\forall x \in \mathbb{R}$  find  $g \circ f, f \circ g, f \circ f, g \circ g, f \circ h, h \circ f, f \circ h \circ g$ . (07 Marks)

**Module-4**

- 7 a. How many integers between 1 and 300 (inclusive) are
- i) Divisible by atleast one of 5, 6, 8
- ii) Divisible by none of 5, 6, 8. (07 Marks)
- b. Find the rook polynomial for the  $3 \times 3$  board by using the expansion formula. (07 Marks)
- c. Solve the recurrence relation  $a_n - 3a_{n-1} = 5 \times 3^n$  for  $n \geq 1$  given that  $a_0 = 2$ . (06 Marks)

**OR**

- 8 a. The number of virus affected files in a system is 1000 (to start with) and this increases 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. (06 Marks)
- b. Solve the recurrence relation  $a_n = 2(a_{n-1} - a_{n-2})$  for  $n \geq 2$  given that  $a_0 = 1$  and  $a_1 = 2$ . (07 Marks)
- c. Compute derangement of  $d_4, d_5, d_6, d_7$ . (07 Marks)

**Module-5**

- 9 a. Define Isomorphism. Verify the given two graphs are Isomorphic (Fig.Q.9(a)). (07 Marks)



Fig.Q.9(a)

- b. "A tree with 'n' vertices has n - 1 edges". Prove this. Define a tree. (06 Marks)
- c. Construct an optimal prefix code for the given set of frequencies, 20, 28, 4, 17, 12, 7. (07 Marks)

**OR**

- 10 a. Explain complete graph, Bipartite graph, subgraph, regular graph, spanning subgraph, minimally connected graph, with example for each. (07 Marks)
- b. Apply merge sort to the given list -1, 7, 4, 11, 5, -8, 15, -3, -2, 6, 10, 3. (06 Marks)
- c. Obtain an optimal prefix code for the message "LETTER RECEIVED" indicate the code. (07 Marks)

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